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The paper divides into three parts: *first*, an analysis of the role of mathematics and mathematicians over the centuries; *second*, specific consideration of the situation in the Philippines and at the Ateneo de Manila; *third*, an outline of a program of action (for mathematics and mathematicians in the Philippines).

I. ANALYSIS OF THE ROLE OF MATHEMATICS AND MATHEMATICIANS

This somewhat lengthy section is necessary because the discipline is much misunderstood today (even by mathematicians themselves). Moreover, there has been a lot of soul-searching over the last couple of years on how mathematicians should be trained and what they ought to be doing (for proof, pick up any recent issue of the *Notices of the American Mathematical Society* or the *American Mathematical Monthly*). At this point, a definition of mathematics is probably expected; unfortunately, none that is satisfactory exists, the best perhaps being that: "Mathematics is what mathematicians do." While logically unacceptable (because circular), it has a great deal of truth in it. When one speaks of what music is, one turns to the work of the masters — a Haydn, a Mozart, a Beethoven. When one speaks of mathematics, one turns to . . . And here we run into a problem. For while everyone knows a great musician or two, and the educated public knows some great physicists (Einstein, Bohr, Oppenheimer, Heisenberg), few know any great mathematicians. A few may remember von Neumann because of his work on the atomic bomb, some may have heard of Riemann (thanks to Einstein's use of Riemannian geometry) — but who has heard of

Gauss, Galois, Poincare, Hilbert? It may be the fault of Alfred Nobel who for reasons known only to himself (some say it is because he could not stand Mittag-Leffler, a mathematician of the time) did not institute a Nobel prize for mathematics (lately, a Nobel prize has been added for economics and ironically the prizes have been going to mathematical economists). In any case, we see how little is known of what mathematicians do (and therefore of mathematics). What will be done here is simply to point out some things that mathematicians do relevant to the understanding of their role in the country today.

General context. What do mathematicians do? The mathematician tries to understand reality. So, of course, does every thinker. And it is worthwhile noting that once upon a time every thinker tried to understand all of reality. The day is long past when a philosopher could write outside his door: Let no one ignorant of geometry enter, — the search for deeper understanding has forced us to specialize. And yet reality remains one — and we are trying once more to see the totality beyond our specialization, not as one-man universal geniuses but through inter-disciplinary teams. This search to understand reality as a whole presents three aspects of the effort of understanding:

(i) Understanding in order to control (or at least predict)

(ii) Understanding to meet man's restless urge to see the meaning of it all.

(iii) Understanding as wisdom.

The mathematician's search to understand falls mainly in the first two categories, but he cannot escape some effort to be wise as well.

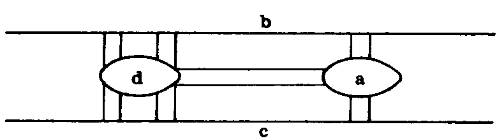
Historical digression. Let us consider two examples from the history of mathematics. Arithmetic and Geometry began quite pragmatically among the Babylonians and Egyptians to meet problems of commerce and land measurement. Methods and tools were, of course, rather *ad hoc* and there was no general theory. But they sufficed to meet the problems of the day. These early mathematicians fall thus into the first category — they tried to understand how numbers and angles and lines behaved to solve their problems (some not quite trivial as manifested by their impressive feats of engineering). It took the Greeks to be fascinated with numbers and angles and lines as such. Arithmetic and Geometry took on a life of their own. While the Egyptian mathematicians had sleepless nights worrying if their pyramids would topple, the Pythagoreans began to be tortured by the irrationality of π and the square root of two. They began to search to understand (mathematically) in the sense of category (ii). We see, then, how these areas of mathematics had their roots in real life problems, but then because of the fundamental urge of the human mind to understand — even its own abstract contructs — they took on a life of their own and were studied for their own sake. We have the beginnings of the distinction between pure and applied mathematics. But the studies of the Greeks were not just ivory tower musings. We owe the arithmetization of the continuum and thence the calculus to their understanding of rational and irrational numbers. And of course our debt is heavy to Euclid's geometry (think of all those parabolic arches). Actually, Pythagorean mathematics sought to understand numbers in order to control the world in a mystical sort of way. It was felt that numbers ruled the world — one sees this in the terminology: perfect numbers, amiable numbers, irrational numbers, in the fascination with the *golden ratio.* One recalls Augustine's ecstatic (number-theoretic) disquisition on the 153 fishes of John 21:11 ($153 = 1+2+3+ \dots +17$ and 17 is a quasi-mystical number) and his admonition: "Therefore, the ravings of Mathematicians are to be treated with scornful laughter — when we have exposed their vain imaginings, made to cast men into the same error into which they have first fallen." (Letter to Januarius). The point is made here to emphasize that even the pure mathematics of the Pythagoreans sought to impinge (if in rather mystical fashion) on reality.

Our second example comes from the development of the calculus. As developed by Newton and Leibniz and used by their contemporaries and immediate successors, it was mainly a brilliant *tool* for physics. In fact, many generalizations from their formulation were simply wrong. A more rigorous formalization had to wait for a later day. At this point of the development of mathematics, the distinction between pure and applied mathematics became clearer. But the distinction between pure and applied mathematician did not obtain. The great names — Newton, Leibniz, later Euler, Gauss — were very competent in Natural Science as well. Their search to understand and control phenomena produced brilliant mathematical constructs and their study of these constructs led to a deeper understanding of phenomena. It is not that the applied work always preceded the purer theory. A case in point is the development of the theory of functions of a complex variable. It was only later discovered that "imaginary" numbers have an amazing role in the formulation of the behavior of electrical phenomena. However, in this classical period of mathematics, mathematicians kept in very close touch with other scientists.

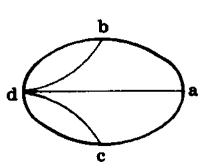
It is a modern phenomenon to have mathematicians forming a ghetto, uninterested in the work of thinkers in other fields — uninterested at least in the sense of not trying to contribute to that work. The signs of the times seem to imply that the withdrawal of mathematicians into pure mathematics, that the present chasm between pure and applied mathematics (one mathematician writes that when he asked his chairman at the University of Zürich some twenty years ago to write his thesis in applied mathematics he was told bluntly, "You are too intelligent to write a dissertation in applied mathematics.") will be short-lived. Be that as it may, mathematics today suffers from this break with problems of the real world — problems being confronted by physicists, biologists, economists — and what makes this of great importance is that the "new" mathematics now being brought into the schools reflects this ghetto mentality of modern mathematics: almost total pre-occupation with axioms, theorems, etc. The "new" mathematics certainly does not reflect the opinion of John von Neumann that: "The most vitally characteristic fact about mathematics is, in my opinion, its quite peculiar relationship to the natural sciences, or more generally to any science which interprets experience on a higher than purely descriptive level." (Thus his own essays into mathematical economics). This is extremely unfortunate in view of the complex problems that face our modern world and the possible contributions that highly-trained mathematicians could make to their solutions (witness the birth of Operations Research when mathematicians were drafted to work perforce on such problems during the Second World War). An atmosphere which shows disinterest if not outright disdain for applications will hardly produce such mathematicians. (This disdain is exemplified by G. H. Hardy, who wrote in his "Mathematician's Apology" that the only great mathematics is useless mathematics. Not completely disdain, it was a reaction as well to such frightening usefulness as the atomic bomb).

Understanding in Order to Control

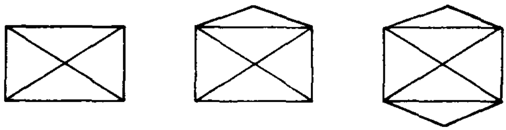
In this particular area, how does the mathematician do his thing? We may use as a paradigm, Euler's solution to the problem of the Seven Bridges of Königsberg. The story is set in the year 1736 in Euler's hometown of Königsberg. The river Pregel runs through the town and there was a network of seven bridges as follows:



In the days before movies and television, the favorite pastime was the evening promenade through the town. It soon became a challenge to try to find a route that would pass through all the bridges exactly once. As all attempts failed, the townspeople decided to write their town mathematician, who was then court mathematician to the Empress of Russia, for a solution. We do not know how long it took Euler to think about the problem. But in the "solution" he sent back, he went as follows: Think, he said, of the land masses *as if* they were points, and the bridges *as if* they were connecting arcs. The diagram then takes on the following form:



The problem of finding a route crossing each bridge exactly once is then the same as the childhood puzzle of drawing the above figure without retracing a line or lifting the pencil. On the way to understanding the problem, consider similar diagrams such as:



A little trial and error shows that (b) and (c) yield a solution and (a) does not. Also that in (b) one has to start at one of the *dark* points and end at the other. In (c), one can start anywhere and end at the same point. We cannot pursue the analysis any further, but Euler finally showed that the important point is to look at each of the vertices and count the number of arcs coming out of each vertex. If *all the numbers are even* as in (c), one can trace the diagram beginning at any vertex and ending at the same vertex; if *exactly two are odd* as in (b), one must start at an odd vertex and end at the other; if *more than two are odd* as in (a), then the diagram cannot be traced. Returning to the diagram associated with the Königsberg bridges we see that *all four vertices are odd* — so there is no solution to the Königsberger's promenade problem.

We see the steps in Euler's understanding and solution of the problem:

(a) He first constructs a model (the "as if" part) — a model which captures the essence of the problem and yet is simpler and more manageable.

(b) He then experiments with variations on the model to find out what properties determine the solution.

(c) He then formulates a hypothesis (probably several hypotheses which had to be refined and corrected) which when tested proves to be correct.

(d) He gives a proof of why the hypothesis in (c) determines the solution.

Another example may be afforded by the work of a British biologist drafted to study the optimum use of planes by the

RAF. At that time the RAF kept half the planes in the air and half on the ground (for maintenance, repair, etc.). The closest model he knew was the flight behavior of birds — amazingly the model fitted and he was able to show that the RAF could keep 80% in the air and 20% on the ground without long-term loss. The models a mathematician is familiar with are abstract constructs:

> Twenty-five years ago, sociologists C. Levi-Strauss and mathematician A. Weil used the theory of finite groups to classify Australian aboriginal clans by marriage and kinship structure. More recently, Hoffman applied classical methods of Lie groups and Lie algebras to develop preliminary models for certain biological-psychological phenomena of visual perception. Modifications of Hoffman's basic approach may help to explain such things as the regional development pattern of urban communities and some features of social and psychological interaction. A paper by three British mathematicians shows that the abstract concepts of algebraic topology (similar to Čech cohomology and relative cohomology) can be employed to describe certain aspects of the planning of urban communities. (Dale W. Lick, New Directions and Commitments for Mathematics, Notices of the American Math. Society, October, 1972, p. 272).

The abstract models above (finite groups, Lie groups and Lie algebras, Čech cohomology and relative cohomology) are, unfortunately, too sophisticated to be described here. Coming back to more familiar ground, algebraic solutions to word-problems fall into the same pattern — we essentially construct an abstract model involving equations in x's and y's.

What is important about this "as if" method of thinking is, of course, that:

(a) the models capture the relevant parts of the concrete structure;

(b) they be manageable and give hope of a solution.

The models a mathematician constructs (and mathematical economists, too, as far as I can gather) will usually have (b) but often fail in (a). The scientists closer to the problem will construct models satisfying (a) (often not much more than a description of the phenomena) but often fail in (b).

Understanding the Meaning of It All

Euler's solution to the Königsberg bridge problem soon began to be studied abstractly — and systems of vertices and interconnecting arcs (now called graphs) were meditated upon for what structure and properties they might possess. The *Theory of Graphs* was born and we see where models constructed to shed light on a given reality begin to take on a life of their own. This study of vertices and arcs has led to applications as well, e.g., a study of freight-train scheduling via these methods has led to impressive simplifications and savings in the railway system of France. The point of interest here, however, is to understand somewhat the rationale of studying these abstract constructs in themselves, the rationale of pure mathematics.

The pure mathematician's reaction to the question of the rationale of studying pure mathematics would be, of course, that it needs no justification. "A thing of beauty is a joy forever." And pure mathematics is beautiful, with an austere, black-and-white sort of beauty. One may choose, therefore, to study mathematics simply as an art. But if one studies it with a view of coming to a deeper understanding of larger reality what is the point of the long hours meditating on Pure Forms — Forms without clear contact with reality? *First of all,* of course, because such abstract forms do have contact with reality. The study of the structure of groups-in-themselves has merit, not just because it is fascinating, but because it will shed light on the many group-like structures in the real world. *Second,* because it is through the study of pure mathematics that one learns to look, like an Euler, at the map of the Königsberg bridges and abstract a graph. One learns a sense of (mathematical) simplicity and elegance — the sense which is usually one's surest guide to a correct formulation and solution. Not to be distracted by irrelevant detail, but to go to the heart of a problem. The difference between a seasoned mathematician and a novice is often that the former will instinctively avoid many wrong paths and will sense quickly when something is going wrong, whereas the latter will go in circles and has no sense of what approaches are promising and what are not. The long hours meditating on the meaning of abstract structures (I spent six months once trying to see my way through one such structure and finally had to give up in defeat) is thus necessary if one is to be effective in providing worthwhile abstract models for reality. The mathematician in real life has to be *simul in actione contemplativus* — there is need of the complementarity between grappling with reality and withdrawing into solitude to try to understand.

Mathematics and Wisdom

Mathematical training cannot give wisdom; no abstract discipline can. Wisdom is learned through coping with the finiteness of one's existence, through suffering and the experience of limitation. But the mathematician pondering the overwhelming problems of the day, the non-existence of ready solutions, mathematical or otherwise, should become conscious of his limits, of the limits of his discipline (and of all disciplines) and break out of his isolation into a common effort with other thinkers and workers. More than that, he must accept that science and all efforts of the mind cannot bring utopia — in fact, our present ecological problems, our atomic arsenals, the Great Wars and the continuing state of war point up that knowledge does not always lead to the good, knowledge is not wisdom, and we must begin to cope with that part of the world equation scientists have been loath to admit as having relevance — the mysteries of freedom and evil in the human heart. We are learning wisdom through suffering — through the common experience of powerlessness in the face of the massive problems of today's world.

II. THE PHILIPPINE SITUATION

We are becoming aware today of the general invalidity of universal solutions to concrete problems. This is certainly being recognized in the field of economics as it becomes clear that Western models of growth are not applicable to the third world. We are conscious of the "situatedness," the "rootedness" of all that we do and think and are and strive for. In seeking the role of mathematics and mathematicians in the Philippines, therefore, we must first understand our specific situation. What is it that we are trying to understand and control? What are we seeking manageable models for? Our struggle is with:

(a) Nature — dire weather, crop pestilence, forest replenishment, etc.

(b) Complex (largely man-made though interacting with nature) Systems — urban housing, transportation, economic structure, etc.

(c) The Human Variable — the imponderables of the human mind and heart.

A. Nature

The struggle to understand and control natural phenomena is largely the task of the natural scientists: the biologists, chemists, physicists. If one studies the history of science it is certainly clear that modern mathematics has not made contributions to science comparable to what it made before the turn of the century. This is largely due to its withdrawal into the isolation of pure mathematics. A change is perceptible now (witness the numerous symposia in matheamtical biology, etc.), but the dialogue and interdisciplinary effort must be accelerated and intensified. Mathematical training in the Philippines, following the U.S. model, has put no premium on competence in another (physical) science. This should be corrected — and the young mathematicians should have competence in one or another natural or social science. Interdisciplinary teams should be formed — dialogue should be initiated with the various scientific societies and within the divisions of the National Research Council. One would hope that mathematicians can make contributions such as those of Charles Steinmetz in the U.S. at the turn of the century when he trained electrical engineers in the use of complex quantities in alternating-current theory. (Quoth one author, "... the vast majority of electrical engineers found it incomprehensible, and were completely mystified that the square root of minus 1 should have anything to do with electric currents.") The Hardy-Weinberg law in population genetics is mathematically trivial (once one thinks about it), but it would never have been formulated (at least not by Hardy) had not Hardy known some basic genetic theory. If mathematicians are to contribute to the struggle with nature they must have some understanding of the struggle — and so learn something of the worries of their scientist colleagues.

B. Complex (Man-made) Systems

During the second world war, large groups of mathematicians and scientists were put to work on solving the complex problems of organization and production that the war effort required. Thus was born the beginnings of a systematic analysis of solving problems inherent in the coordination of complicated interlocking systems: called Systems Analysis in Great Britain, Operations Research in the U.S.A. (Wars, like typhoons with the soil and the sea, have a way of bringing out creativity as well as destruction.) After the war, large corporations such as Bell Telephone, IBM, Rand, etc. employed these "applied" scientists — and the scientific approach to solving business problems was initiated. Lately, governments have come to recognize the possibilities in such systematic approaches to the complexities of government planning. We should have no illusions about the scope and efficacy of present-day mathematical efforts to systematize, say, economic planning. Available techniques are certainly no better than the horrendous *ad hoc* methods natural philosophers had for studying physical phenomena before the advent of the calculus. The models of the economy one can come out with are more likely to resemble the Ptolemaic model of the heavens with its epicycles upon cycles than the simple and elegant models of Copernicus and Kepler. Often the role of the mathematician in an interdisciplinary team working on a model for the economy or for urban planning can be little more than to demand more precise definitions of terms and rigorous logical analysis — this is due to the limitations of his own available tools as well as the rudimentary state of the preliminary analysis of the economic or urban situation. Still, such a limited role is not to be thought of little account. Much progress can be achieved in any of our planning efforts by just some clearer and more precise thinking — the kind of thinking that keeps its goals clearly in sight and refuses to be distracted by side problems or satisfied by merely apparent solutions. Again, of course, the mathematician should develop some interest and competence in social problems — to be able to function effectively as part of a team. In this area specially he should be conscious of the complexities of the problems, of the human dimension, of the limitations of his techniques — but also of the importance of his serious efforts because of the import of the problems he is faced with. There is particular need in a country like the Philippines for roles such as these for mathematicians: for, on the one hand, one finds so much shoddy conceptualization going for planning, on the other hand, many planners are over-awed by mathematical technique applied, say, to decision-making and attribute to them magical properties (we are not really so far from the magician-mathematician views of the Pythagorean age), instead of realizing that they are usually little more than disciplined common sense. (As a friend has remarked, the tragedy is that if one has common sense, one has relatively little need of such techniques; and if one does not, no technique can supply for it.) Anyway, it is easy for our (say, educational) planners to be over-awed by foreign advisers — and swallow such (expensive) non-solutions as more gadgets and novel methods of instruction and fail to come to grips with the roots of the problems. If the mathematician on a planning team cannot keep his head when a brilliant statistical analysis is presented showing how the latest gadget has raised IQ's by an average of 10 points, pity the poor educator! (E.g., at the root of our educational problems in the Philippines is a philosophy of mass education we inherited from the U.S. but unfortunately cannot support. No amount of hocus-pocus with new gadgets or methods can make this problem go away. It has to be met head-on.) It is strange, perhaps, that an important role for the mathematician is to demythologize mathematical technique.

C. The Need of Wisdom

As we move from the control of natural phenomena to social problems, the human variable enters our equations — and unfortunately it is not readily tractable by scientific technique (this, notwithstanding B.F. Skinner). The American saying that "If we can go to the moon, we can solve our slum problems," is really a fallacy — the problems involved belong to different orders. The mathematician must realize that in the face of problems such as the Muslim-Christian conflict, clear and rigorous thinking remains woefully inadequate. One has to learn wisdom — and part of that wisdom is to learn the limitations of any solutions. It is important for a mathematician who participates in planning to learn this wisdom — for otherwise he will forge nice, clean plans, which unfortunately promptly fail. Because of the stupidity of the people he will say, not realizing that this human variable should have been taken into account and cannot be used as an alibi for failure.

III. A PROGRAM OF ACTION

How are mathematics and mathematicians to assume something of the roles outlined above? The task is two-fold:

(a) to specify directions of effort — in interdisciplinary effort with other thinkers and workers;

(b) the training of mathematicians with the requisite vision and skills. (Where are they to come from?)

The problem of the U.S. seems to be largely one of misplaced or mistrained mathematicians. *We* have few mathematicians, mistrained or otherwise. The re-direction of effort and the training of qualified people have to go hand in hand.

A. Present (and Recent Past) Efforts (National Level)

The Mathematical Society of the Philippines was organized recently to provide an organization to systematically carry out programs which would lead to the above goals. Last year I ran a seminar on Graph Theory for the Ateneo math faculty in order to initiate study of new directions in mathematics. We shall now run such seminars on a wider basis. The objective is to produce a common effort among Filipino mathematicians (at least in the Greater Manila area) towards relevant directions of teaching and research. Behind this effort is the continuing conviction that it is an organized group of men sharing some common vision that will bring about change.

B. Efforts (Ateneo Level)

(a) The Ateneo Math Department has been designated a Graduate Center by the FAPE (Fund for Assistance To Private Education). We now have several scholars working for the M.S. and their program is designed to help achieve some of the above goals.

(b) On the undergraduate level, the Management Engineering Program is designed to be precisely an interdisciplinary program with strong quantitative emphasis. The Human Development Committee has been working with it as the pilot program—ideally, its graduate should be well equipped to cope with problems of planning and management in business and government.

(c) In line with the above analysis, the B.S. Mathematics major should develop some competence in a natural or social science.

(d) So much of what is written above is concerned with the professional mathematician. How about everyman and the mathematics he learns or should learn? We are beginning to try to develop basic courses that better reflect the role of mathematics and mathematicians outlined above. Among other things, trigonometry should be dropped (except for an understanding of trigonometric functions) and some rudimentary knowledge of probability, statistics, and matrix theory should be brought in as being necessary for everyman's understanding of the modern world. Some topics from, say, the mathematics of decision-making should be included to give the student an insight into present mathematical efforts. The Mathematics Department is now working on material for such new courses.

C. Material Output

More specific products for the HDC project will be course/ seminar material on two levels:

(a) Material for a Freshman course embodying the above ideas and goals.

(b) More advanced material for faculty, graduate students, and advanced undergraduates — some is already available in graph theory; we shall be working on other topics over the next semester.

IV. CONCLUSION

We live in a complex world and the solution to its problems demands a complex, but united effort. We must really see the tasks outlined above as aspects of one task (which, again, is but one part of the interdisciplinary effort). The task of producing mathematicians who can contribute effectively to interdisciplinary teams has the same goal as the task of producing a freshman math course which properly situates mathematics in the student's training and world-view. The common goal is to have mathematics play its proper role (modest or otherwise) in the task of understanding and coping with reality — a reality that threatens at times to overwhelm us, thus making this effort something more than an academic exercise.