philippine studies

Ateneo de Manila University • Loyola Heights, Quezon City • 1108 Philippines

Mathematical Research in the Philippines

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Philippine Studies vol. 15, no. 2 (1967): 241–258

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F. M. SIOSON

BY the very nature of mathematics, mathematical research, whether it be pure or applied, may always be considered basic. Its conclusions are highly abstract and cover an unusually wide range of applicability and interpretation. Its results are rigorous and of a character that is indisputable and permanent. A researcher in mathematics derives satisfaction from discovering new knowledge as well as from composing structures and forms of pure thought endowed with beauty all their own. Nowhere in the whole domain of human lore can one find a more rewarding and more honest encounter between man and his own intellect. Mathematics is not only the queen of the sciences, but also a fine art, and above all, one of the greatest of the classical and modern humanities.

A practitioner of the science and fine art that is mathematics is often referred to as a mathematician. Many seem to agree that this simply means an individual who in the past has contributed something new to existing mathematical knowledge. Under this criterion, the possessor of a Ph.D. degree in mathematics is generally considered to be a mathematician, but this is by no means sufficient. There are mathematicians who never get advanced degrees. For this reason, others may prefer other criteria. The International Mathematical Union, for instance, would list anybody in the

World Directory of Mathematicians only if he has at least two papers reviewed by one of the three existing reviewing journals of mathematics: the *Mathematical Reviews* in America, the *Zentralblatt für Mathematik und ihre Grenzgebiete* in Western Europe, and the *Referativnyi Zhurnal Matematika* in the Soviet Union. This is rather an exacting criterion, for not all mathematical journals are noticed by these reviewing agencies, not all doctoral dissertations in mathematics are publishable or ever published, and only a very small fraction of mathematical doctors continue to produce after writing their dissertations.

Under any criterion accepted by the mathematical establishment, there appears to be no record whatsoever of any Filipino who attained the status of a mathematician throughout the three hundred and fifty years of Spanish colonization in the Philippines. Elementary mathematics was taught in the local colleges and in the only university at the time and a number of textbooks in mathematics appeared in Manila during the eighteenth and nineteenth centuries. A partial list of such books may be found in Eulogio B. Rodriguez's "Brief Observations on Science in the Philippines in the Pre-American Era", *Encyclopedia of the Philippines*, (Manila, 1957) 13, 39-88: among these are Ignacio Villamor's *Arithmética Elemental* and *Geometría Elemental* which were both published in the year 1897. However, to the best of my knowledge, I am not aware of any original mathematical paper or monograph that ever came forth from the various centers of learning in the Philippines during those years. And even among the handful of European-educated Filipino intellectuals who flourished during the waning years of Spanish rule and the early years of American occupation, mathematical research was a totally neglected and unknown vocation.

The first creative efforts in mathematics by Filipinos did not really appear until the early twenties of the present century. A master's degree program in mathematics was introduced in the University of the Philippines at that time. The first graduate of this program is Francisco D. Perez who

obtained his M.A. in 1924 under the tutelage of an American professor, H. L. Smith. His master's thesis [44] was a study of the derivative of an algebraic function without the use of the notion of limit. It was a nice piece of work.

The first Filipino holder of a Ph.D. degree in mathematics is, however, Emeterio Roa. He graduated from the University of Michigan in 1923 with a dissertation entitled: *A Number of New Generating Functions with Applications to Statistics.* Utilizing methods previously proposed by his academic mentor, Professor H. C. Carver ("The mathematical representation of frequency distributions", *Quarterly Publication of the American Statistical Association,* 17 [1921]) and C. V. L. Charlier ("Uber die Darstellung willkürlicher Funktionen", *Arkiv för Matematik Astronomi och Fysik,* 2 [1905] pp. 1-35), E. Roa investigated a number of new generating functions which could be employed in graduating frequency distribution functions.

Vidal A. Tan was next to earn his doctorate in 1925 from the University of Chicago under the guidance of Professor Ernest P. Lane, a well-known differential geometer. A year later, Enrique T. Virata followed with a similar degree from Johns Hopkins University. Both had previously obtained master's degrees, the former from Cornell University and the latter from Harvard. Both also worked in the field of classical differential geometry, which was in vogue during those days. Vidal A. Tan's dissertation [82] was about projective properties of quadrics generated by the osculating conic of a pencil of planes passing through a fixed line tangent to a given surface. Virata's work [97], on the other hand, is devoted mainly to metric differential geometry: he classified certain solutions of the Gauss differential equation. Neither Tan's nor Virata's dissertation has ever appeared in print.

Through the good offices of Vidal A. Tan who was head of the department of mathematics at the University of the Philippines, a very able Indian mathematician by the name of Vishnu D. Gokhale came to the Philippines in 1923 to teach at the state university as an associate professor of mathematics. His training was in algebra, but his interests were wide and varied. He remained in the country until his political

leanings forced him to leave the Philippines some time in the middle forties. Gokhale was Professor and Chairman of the mathematics department in the Negro University of Atlanta when he died in 1956. Despite the many very complimentary reports about his ability as a mathematician, his genius never seemed to have flowered. His doctoral dissertation [14] written in 1922 at the University of Chicago under that famous teacher and mathematician, Professor Eliakim Hastings Moore, was indeed a very elegant piece of research. It was published in the *American Journal of Mathematics* in the same year. Except for this, however, many of his papers are quite inconsequential. These include two notes [15]-[16] published in the *U.P. Natural and Applied Science Bulletin*: the first of which is devoted to the solution of a linear equation in one unknown over the field of quaternions; the second, to the observation that the Cauchy-Toeplitz Theorem also holds for arbitrary valuation (or Kürschak) fields. One of his abstracts [17] appeared in the *Bulletin of the American Mathematical Society* in 1951 and a number of *Annual Reports of the President of the University of the Philippines* noted some results of his researches, but none of these have ever been published, here or abroad. Just before his death, he requested Chio-Shih Lin, who was then in the United States, to transmit to Professor Irving Kaplansky of Chicago the manuscript of a long memoir on Banach spaces for comment and publication, but according to Lin, the latter was not very kind.

In 1929, two more Filipinos were awarded their Ph.D.'s in mathematics at the University of Chicago. The first of them, Francisco D. Perez [45] developed the theory of integral equations of the Hilbert-Schmidt type over the field of quaternions. He was the last doctoral student of Professor Eliakim Hastings Moore. The second, Trinidad J. Jaramillo [20] worked on the elasticity theory of bodies whose stress-energy function is homogenous and quadratic in the first and second order partial derivatives of the components of displacement with respect to the various spatial coordinates. A year later, in 1930, Luis R. Salvosa followed suit by getting his Sc. D. degree in mathematical statistics from the University of Michigan. His dissertation [54] consisted mainly of the preparation

of a complete set of standardized tables which give the ordinates, areas, and derivatives of Pearson's type III curve for intervals of 0.01 in the argument and 0.1 in skewness. His tables were published in the first volume [55] of the *Annals of Mathematical Statistics.*

At no other time than during the early thirties, in the whole history of mathematics in the Philippines, was there a more opportune moment to build a mathematical tradition in our country. Certainly, we had a suitable academic climate then and the potential human resources. Many of our Filipino doctors of mathematics then had studied under very distinguished men like E. H. Moore, Leonard E. Dickson, E. P. Lane, W. C. Graustein and others, who were leaders in the development of the great mathematical tradition that now exists in the United States. What happened, however, was that instead of further cultivating and developing their talents through zealous and sustained research, many of these men chose to divert their energies and efforts to various economic, political, and administrative pursuits.

This group at the State University was further strengthened in the thirties by the influx of younger men like Chio-Shih Lin, Leopoldo V. Toralballa, and Raymundo A. Favila.

Chio-Shih Lin, before coming to Manila, was an instructor of wide experience at the University of Amoy. He came to the State University to work for his master's degree, which he obtained in 1931 under Dr. Francisco D. Perez. Subsequently, Lin settled down in the Philippines and has since been in continuous residence as a professor at the University of the Philippines, except for a short stint at Ohio State University where he obtained his doctorate in 1955 under the well-known Austro-American number theorist Henry B. Mann.

Leopoldo V. Toralballa is a local talent. He obtained both his bachelor's and master's degrees from the University of the Philippines. Not long after that, he went abroad for further graduate work and earned his Ph.D. in 1941 from the University of Michigan. His dissertation [87] was concerned with

the summation of values of rational functions which are allowed to vary over all n-tuples of natural numbers whose total is m. He came back to Manila for a while, taught at the Far Eastern University and together with T. J. Jaramillo worked as actuary in an insurance company in the city. Eventually, however, both went back to the United States: Jaramillo to the Midway Laboratories in Chicago and Toralballa to Marquette University in Wisconsin. Toralballa is at present an Associate Professor of Mathematics at New York University and Jaramillo, a Mathematical Consultant at the University of Chicago Computation Center. In addition to his master's [85] and doctoral [87] theses, L. V. Toralballa has published a couple of very short notes [86] and [94] in the *American Mathematical Monthly*. He has announced about a half-dozen abstracts in the *Bulletin of the American Mathematical Society* [88]-[93] and one in the *Proceedings of the International Congress of Mathematicians in 1950* at Cambridge, Massachusetts [95], but the details of these researches have never been published in mathematical journals.

In contrast to the preceding two, Raymundo A. Favila obtained his mathematical education entirely in the United States. He earned both his bachelor's and doctorate degrees from the University of California at Berkeley, the latter in 1939. He is another differential geometer. His doctoral dissertation [10] deals with the theory of linear congruences, particularly, with pairs of linear congruences whose generators are in a one-to-one correspondence. After teaching for a year at the University of California at Berkeley as an instructor, Favila joined the faculty of the University of the Philippines through the invitation of Dr. Vidal A. Tan. At present, he heads the department of mathematics there.

Among these first Filipino doctors of mathematics, Dr. F. D. Perez has, in my opinion, the best record as a creative mathematician. Notwithstanding his poor health, his heavy duties as a professor, writer of elementary mathematical textbooks, assistant head and later head of the department of mathematics at the University of the Philippines, he managed to publish, off and on, mathematical papers of good quality

through the years. All of his published papers [46]-[49] appeared in the *U.P. Natural and Applied Science Bulletin.* They all deal exclusively with the theory of matrices and determinants and their generalizations. In many respects, Dr. Perez has never rid himself of the influence of that great teacher E. H. Moore. This influence seems also to pervade the work of his four thesis students: Chio-Shih Lin [26], S. J. Estrada [9], Matias Arreola [6], and Felina Mapa [30].

The years after Favila were lean mathematical years. No new Filipino Ph.D.'s in mathematics were produced during that period. The lull was finally broken in 1958 when Cristina P. Parel received her doctorate in mathematical statistics from the University of Michigan with a dissertation entitled: *A Matrix Derivation of the Generalized Least Squares Linear Regression with All Variables Subject to Error.*

The author followed in 1960 with a degree in algebra from the University of California at Berkeley. The author's dissertation [59] and first published papers [60]-[62] were devoted to the theory of primal and independent algebras. His contact as a student, however, with the logic and methodology group at Berkeley has left in him an abiding interest in logic and Boolean algebra. Sioson has published a number of papers [67], [68], [69], [71], [72], [74], [80] on these and other related subjects. His sojourn at the University of Hawaii in 1963-1964 opened even wider mathematical vistas for him. Here the fast-growing field of semigroup theory came to his attention. In the ensuing months and years, the author has made some modest contributions [63], [64], [65], [66], [73], [78], [81] to this subject. He has particularly shown that certain fundamental results in the theory of semigroups may be extended to more general systems called m-semigroups. Some of the author's results in this direction have since been generalized and unified by a Russian mathematician, L. M. Gluskin ("O positzionnih operativah", *Doklady Akademiia Nauk SSSR*, 157 [1964] 767-770; "Positzionnih operativah", *Matematicheskii Sbornik* 57 [1965] 444-472). His interest in general algebraic systems has wavered, but it has never been lost, for, during the last seven years, he has

continued to contribute to this field [61], [70], [75], [76], [79]. Through the invitation of Professor Alexander Doniphan Walace, a leading world authority on the subject of topological semigroups, F. M. Sioson joined the mathematics faculty of the University of Florida in 1964. Here, he was confronted by the traditional mathematical strength of the American South: point-set topology. Inspired by his newly acquired understanding of this fascinating subject, the author introduced the notions of topological m-semigroup [73] and topological m-group. The latter notion was the subject of the dissertation of Sioson's first Ph.D. student.

The establishment of the Statistical Training Center in Manila during the early fifties has necessitated the training of a number of statisticians with doctorate degrees. Some of these individuals have delved into research activities that are of mathematically creative nature, but many of them are mere users of mathematics. A good example of statistician of the former type is Dr. Tito A. Mijares, now the Director of the Bureau of the Census and Statistics. He is one of the first to finish in the graduate program of the Department of Statistics at Harvard. He has published at least two papers in the *Annals of Mathematical Statistics*, both dealing with the distribution of certain functions of the eigenvalues of matrices. The prime interest, on the other hand, of the latter type of statisticians is confined not to the development of new statistical tools and techniques, but to the application of known statistical methods to the various sciences like biology, agriculture, business, economics, and others. These people are adequately trained to analyze statistical problems covered by known standard results, but they are often unable to devise methods to handle new situations. Their researches cannot rightly be called mathematical.

The roster of Filipino mathematicians has been growing and will continue to grow in the years to come. There are presently about a dozen or so students from the Philippines studying for their doctorates in mathematics in American universities. Hopefully, a greater degree of concentration of doctors of mathematics in our universities may trigger a certain

reaction that can eventually give rise to some form of a mathematical mystique. However, it is fallacious to think that numbers alone can create the tradition that we are looking for. In the experience of many other countries, it takes only two or three determined people who have the ability to make new mathematics to start one. This, of course, cannot happen in an intellectual desert: there must be a climate and environment in which these individuals can thrive, grow, and produce. In the spirit of humility and intellectual honesty, our society should learn to recognize and reenforce mathematical talent.

There is an overwhelming need of developing a Filipino mathematical school. I would like to propose the following as means of achieving this important goal:

(1) Our mathematical efforts and talents should be directed to one field of mathematical learning. This need not be in the mainstream of current research, but it has to be something young, vigorous, and teeming with unsolved and challenging problems. Our resources are limited: it is therefore inevitable that we should concentrate on only one specialty.

(2) The founding of a specialized journal devoted to this one field of mathematical research is equally necessary. If we are to occupy a place in the world of mathematics, we must show our own initiative. Such a journal must be international in scope. To maintain a high quality publication, a very strict referee system should be adopted. Whenever necessary, we should not hesitate to invoke the help of foreign colleagues. Their contributions and support would infuse life and vitality into our journal. Possibly, a journal such as this should be governed by an editorial advisory board consisting of experts in the field the world over.

Almost a half-century has elapsed since the appearance of the first creative attempts in mathematics by Filipinos. Considering our size, our contributions to the international pool of mathematical knowledge have been very meager, but, bearing in mind our age as a nation, they are perhaps adequate. However, to keep intact the faith of our people and friends in the future of science in our republic, we must contribute a

greater share to this pool. It is not a gross exaggeration to say that our readiness to partake in the blessings of the industrial revolution is best measured by our scientific achievements, in general, and by our mathematical achievements, in particular. We cannot be a partner in the entrepreneurship of science by always being a consumer and never a producer.

The following bibliography of Philippine mathematical achievements is fairly complete, except for a few items. It does not include master's theses, doctoral dissertations, and papers which I thought did not contribute anything new to existing mathematical knowledge. It does not mean, on the other hand, that all entries written here are original contributions. Many of them are duplications of known results. There are items I have omitted simply because of lack of further information or details. There are, for example, the master's theses of Roberto S. Mariano and Eugene Santos from the University of the Philippines and the doctoral dissertation of Mrs. Alicia Hidalgo-Santos from the University of Florida last year. Two of Dr. Eutiquio C. Young's abstracts listed hereunder have already appeared in the *Journal of Differential Equations* and the *Journal of Mathematical Analysis and its Applications.*

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